

AMERICAN ACADEMY LARNACA
MATHEMATICS YEAR 6
PLACEMENT EXAMS

Core Mathematics

Advanced Subsidiary

Time: 2 hours

Materials required for examination

Mathematical Formulae (Pink)

Items included with question papers

Nil

Calculators may NOT be used in this examination.

NAME

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 12 questions in this question paper. The total mark for this paper is 90.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.

PLACEMENT EXAMS FOR YEAR 6

1. Find the first 3 terms in ascending powers of x of

$$\left(2 - \frac{x}{2}\right)^6$$

giving each term in its simplest form.

(4)

2.

$$f(x) = \frac{8}{x^2} - 4\sqrt{x} + 3x - 1, \quad x > 0$$

Giving your answers in their simplest form, find

(a) $f'(x)$ (3)

(b) $\int f(x)dx$ (4)

3.

$$f(x) = 10x^3 + 27x^2 - 13x - 12$$

(a) Find the remainder when $f(x)$ is divided by

(i) $x - 2$

(ii) $x + 3$

(3)

(b) Hence factorise $f(x)$ completely.

(4)

4. Answer this question without the use of a calculator and show all your working.

(i) Show that

$$\frac{4}{2\sqrt{2}-\sqrt{6}} = 2\sqrt{2}(2+\sqrt{3}) \quad (4)$$

(ii) Show that

$$\sqrt{27} + \sqrt{21} \times \sqrt{7} - \frac{6}{\sqrt{3}} = 8\sqrt{3} \quad (3)$$

5. A sequence is defined by

$$u_1 = 3$$

$$u_{n+1} = 2 - \frac{4}{u_n}, \quad n \geq 1$$

Find the exact values of

(a) u_2, u_3 and u_4 (3)

(b) u_{61} (1)

(c) $\sum_{i=1}^{99} u_i$ (3)

6. Given that a and b are positive constants, solve the simultaneous equations

$$ab = 25$$

$$\log_4 a - \log_4 b = 3$$

Show each step of your working, giving exact values for a and b .

(6)

7. Find the range of values of k for which the quadratic equation

$$kx^2 + 8x + 2(k + 7) = 0$$

has no real roots.

(7)

8. The first three terms of an arithmetic series are 60 , $4p$ and $2p - 6$ respectively.

(a) Show that $p = 9$.

(2)

(b) Find the value of the 20th term of this series.

(3)

(c) Prove that the sum of the first n terms of this series is given by the expression

$$12n(6 - n)$$

(3)

9.

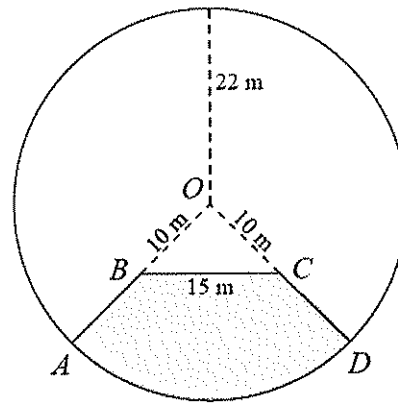


Diagram **NOT**
drawn to scale

Figure 1

Figure 1 shows the plan for a pond and platform. The platform is shown shaded in the figure and is labelled $ABCD$.

The pond and platform together form a circle of radius 22 m with centre O .

OA and OD are radii of the circle. Point B lies on OA such that the length of OB is 10 m and point C lies on OD such that the length of OC is 10 m . The length of BC is 15 m .

The platform is bounded by the arc AD of the circle, and the straight lines AB , BC and CD .

Find

- (a) the size of the angle BOC , giving your answer in radians to 3 decimal places, (4)
- (b) the perimeter of the platform to 3 significant figures, (4)
- (c) the area of the platform to 3 significant figures. (4)
-

10. The curve C has equation

$$y = \frac{(x-3)(3x-25)}{x}, \quad x > 0$$

(a) Find $\frac{dy}{dx}$ in a fully simplified form.

(5)

11.

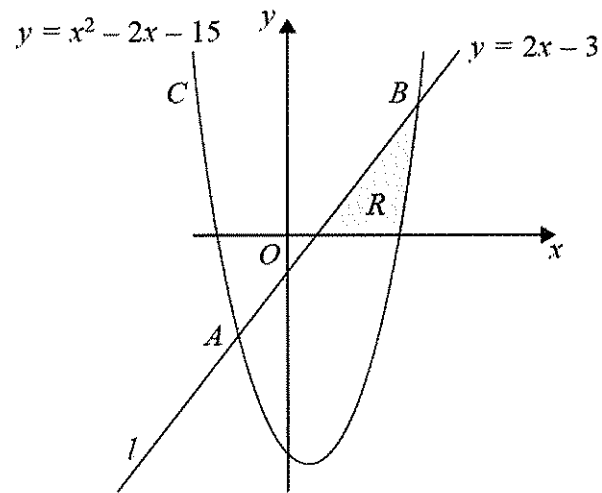


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drawn to scale

Figure 2

Figure 2 shows part of the line l with equation $y = 2x - 3$ and part of the curve C with equation $y = x^2 - 2x - 15$.

The line l and the curve C intersect at the points A and B as shown.

(a) Use algebra to find the coordinates of A and the coordinates of B .

(6)

12.

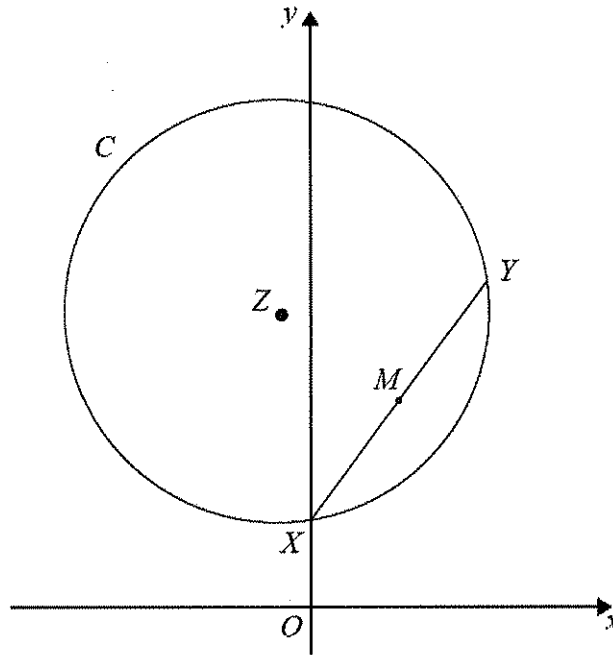


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Figure 3

The points X and Y have coordinates $(0, 3)$ and $(6, 11)$ respectively. XY is a chord of a circle C with centre Z , as shown in Figure 3.

(a) Find the gradient of XY . (2)

The point M is the midpoint of XY .

(b) Find an equation for the line which passes through Z and M . (5)

Given that the y coordinate of Z is 10,

(c) find the x coordinate of Z , (2)

(d) find the equation of the circle C , giving your answer in the form

$$x^2 + y^2 + ax + by + c = 0$$

where a , b and c are constants. (5)

TOTAL FOR PAPER: 90 MARKS

END