AMERICAN ACADEMY LARNACA

PLACEMENT EXAM

Pure Mathematics P1

Sample 1

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Nil	Nil

Calculators may be used in this examination.

Information for Candidates

Full marks may be obtained for answers to ALL questions.
This paper has 9 questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the examiner.
Answers without working may gain no credit.

Question 1.

Given that $2^x = \frac{1}{\sqrt{2}}$ and $2^y = 4\sqrt{2}$,

(a) find the exact value of x and the exact value of y, **(3)**

(b) calculate the exact value of 2^{y-x}. **(2)**

Question 2.

$$f(x) = \frac{(x^2 - 3)^2}{x^3}, \ x \neq 0.$$

(*a*) Show that $f(x) \equiv x - 6x^{-1} + 9x^{-3}$. **(2)**

(*b*) Hence, or otherwise, differentiate $f(x)$ with respect to x. **(3)**

Question 3.

(a) Express $\sqrt{108}$ in the form $a\sqrt{3}$, where a is an integer.

(1)

(b) Express $(2 - \sqrt{3})^2$ in the form $b + c\sqrt{3}$, where b and c are integers to be found.

(3)

(c) Simplify

$$\frac{5 - \sqrt{3}}{2 + \sqrt{3}},$$

giving your answer in the form $a + b\sqrt{3}$, where a and b are integers.

(3)

Question 4.

Find the set of values for x for which

(*a*) $6x - 7 < 2x + 3$, **(2)**

(*b*) $2x^2 - 11x + 5 < 0$, **(3)**

(*c*) both $6x - 7 < 2x + 3$ and $2x^2 - 11x + 5 < 0$. **(1)**

Question 5.

The equation $x^2 + 5kx + 2k = 0$, where k is a constant, has real roots.

(a) Prove that $k(25k - 8) \geq 0$. **(2)**

(b) Hence find the set of possible values of k. **(3)**

(c) Write down the values of k for which the equation $x^2 + 5kx + 2k = 0$ has equal roots. **(1)**

Question 6.

$$x^2 - 8x - 29 \equiv (x + a)^2 + b,$$

where a and b are constants.

(a) Find the value of a and the value of b.

(3)

(b) Hence, or otherwise, show that the roots of

$$x^2 - 8x - 29 = 0$$

are $c \pm d\sqrt{5}$, where c and d are integers to be found.

(2)

Question 7. **Figure 1**

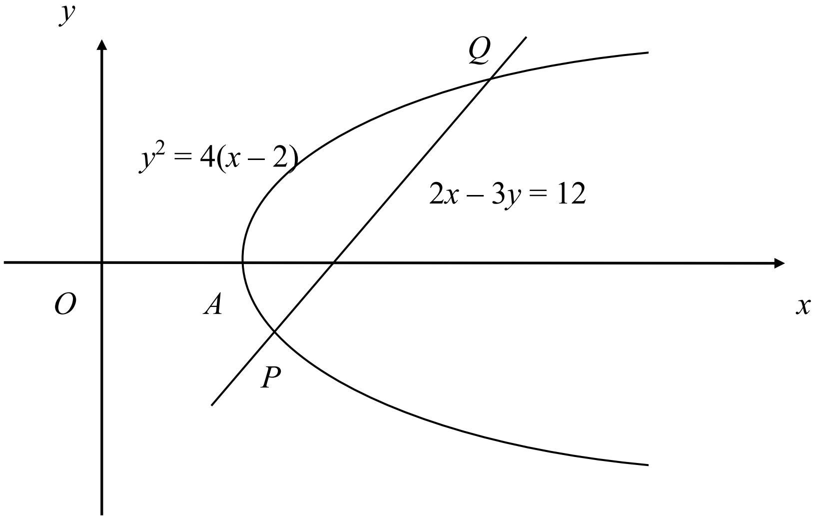

Fig. 1 shows the curve with equation $y^2 = 4(x - 2)$ and the line with equation $2x - 3y = 12$.

The curve crosses the x-axis at the point A, and the line intersects the curve at the points P and Q.

(a) Write down the coordinates of A. **(1)**

(b) Find, using algebra, the coordinates of P and Q. **(4)**

(c) Show that $\angle PAQ$ is a right angle. **(4)**

Question 8.

The points $A(-1, -2)$, $B\ (7, 2)$ and $C(k, 4)$, where k is a constant, are the vertices of $\Delta\ ABC$. Angle ABC is a right angle.

(*a*) Find the gradient of AB. **(2)**

(*b*) Calculate the value of k. **(2)**

(*c*) Show that the length of AB may be written in the form $p\sqrt{5}$, where p is an integer to be found. **(3)**

(*d*) Find the exact value of the area of $\triangle\ ABC$. **(3)**

(*e*) Find an equation for the straight line l passing through B and C. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(2)**

Question 9.

The curve C has equation $y = \mathrm{f}(x)$. Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 20x + 29$ and that C passes through the point $P(2, 6)$,

(*a*) find y in terms of x. **(3)**

(*b*) Verify that C passes through the point $(4, 0)$. **(1)**

(*c*) Find an equation of the tangent to C at P. **(3)**

The tangent to C at the point Q is parallel to the tangent at P.

(*d*) Calculate the exact x-coordinate of Q. **(5)**

Question 10.

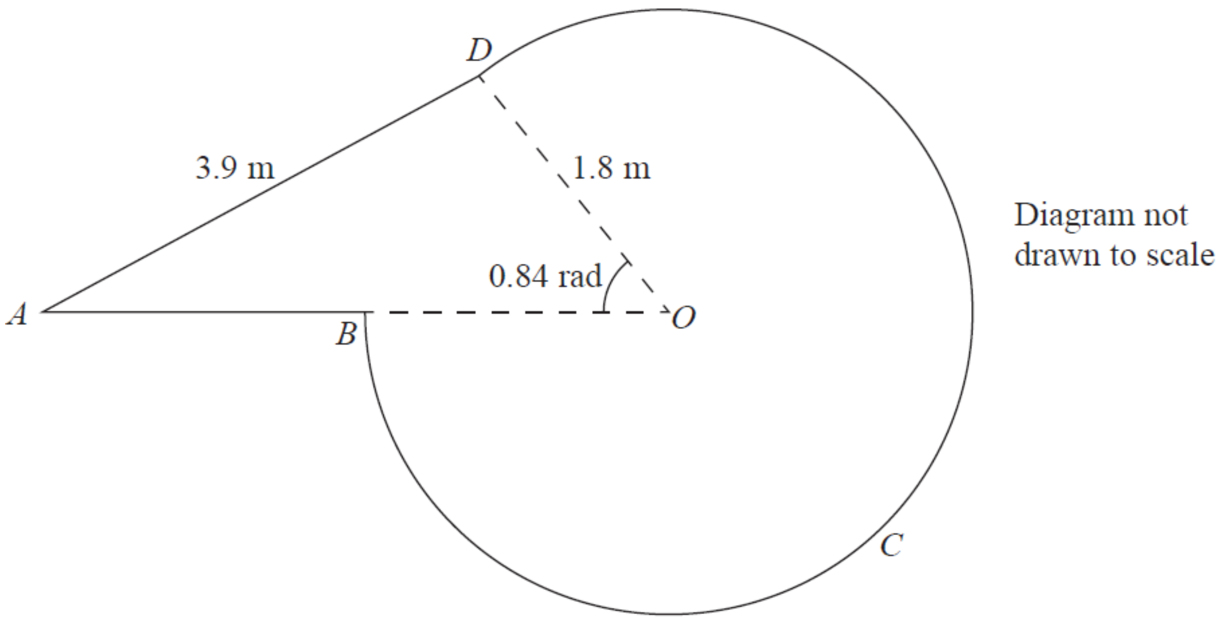

Figure 1

Figure 1 shows the design for a shop sign *ABCDA*.

The sign consists of a triangle *AOD* joined to a sector of a circle *DOBCD* with radius 1.8 m and centre *O*.

The points *A*, *B* and *O* lie on a straight line.

Given that $AD = 3.9$ m and angle *BOD* is 0.84 radians,

(a) calculate the size of angle *DAO*, giving your answer in radians to 3 decimal places.

(2)

(b) Show that, to one decimal place, the length of *AO* is 4.9 m.

(3)

(c) Find, in m^2, the area of the shop sign, giving your answer to one decimal place.

(3)

(d) Find, in m, the perimeter of the shop sign, giving your answer to one decimal place.

(3)

TOTAL: 78 marks

END